

# Noncommutative wormholes in $f(R)$ gravity

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**Abstract:** This paper discusses several new exact solutions of static wormholes in  $f(R)$  gravity with a noncommutative-geometry background, which replaces point-like structures by smeared objects. In the first part of the paper we assume the power-law form  $f(R) = aR^n$  and discuss several solutions corresponding to different values of the exponent. The second part of the paper assumes a particular form of the shape function that also yields a viable solution. This investigation generalizes some of our previous work in  $f(R)$  gravity, as well as in noncommutative geometry.

**Keywords:** wormholes; modified gravity; energy conditions

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## I. INTRODUCTION

Wormholes are topological tunnel-like structures connecting different regions of spacetime. Once believed to be submicroscopic, it was shown in 1988 [1] that wormholes could be large enough for humanoid travelers and even permit time travel. Since then an enormous number of new wormhole solutions and their astrophysical implications in various gravity theories have been proposed. (See, for instance, [2] and references therein.) Most recently, static wormholes have been explored in generalized teleparallel gravity as well [3].

An interesting and important development of string theory is the realization that coordinates determining the geometry may become noncommutative operators in a  $D$ -brane [4, 5]. This results in a fundamental discretization of spacetime due to the commutator  $[\mathbf{x}^\mu, \mathbf{x}^\nu] = i\theta^{\mu\nu}$ , where  $\theta^{\mu\nu}$  is an antisymmetric matrix. There is an interesting similarity to the uncertainty principle in which the Planck constant  $\hbar$  discretizes phase space [6]. This noncommutative geometry is an intrinsic property of spacetime that does not depend on particular features such as curvature. Moreover, it was pointed out in Ref. [7] that noncommutativity replaces pointlike structures by smeared objects, thereby eliminating the divergences that normally appear in general relativity. This smearing can be modeled by the use of the Gaussian distribution of minimal length  $\sqrt{\theta}$  instead of the Dirac delta function. So the energy density of the static and spherically symmetric smeared and particlelike gravitational source has the form [8]

$$\rho(r) = \frac{M}{(4\pi\theta)^{3/2}} e^{-\frac{r^2}{4\theta}}. \quad (1)$$

The mass  $M$  could be a diffused centralized object such as a wormhole [9]. The Gaussian source has also been used by Sushkov [10] to model phantom-energy supported wormholes, as well as by Nicolini and Spalluci [11] for the purpose of modeling the physical effects of short-distance fluctuations of noncommutative coordinates in the study of black holes. Galactic rotation curves inspired by a noncommutative-geometry background are discussed in one of our recent works [12]. The stability of a particular class of thin-shell wormholes in noncommutative geometry is analyzed in Ref. [13].

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Recently Rahaman et al. [14] investigated higher-dimensional wormholes in Einstein gravity with noncommutative geometry. It was shown that wormhole solutions exist in the usual four, as well as in five dimensions, but they do not exist in higher-dimensional spacetimes. Based on that study, Kuhfittig [15] showed that wormholes in noncommutative geometry can be macroscopic. The necessary violation of the weak energy condition is attributable to the noncommutative-geometry background rather than to the use of exotic matter. He concluded that if string theory is correct, then the laws of physics allow macroscopic traversable wormholes with zero tidal forces that are stable to linearized radial perturbations.

In this paper we derive some new exact solutions of static wormholes in  $f(R)$  gravity in a noncommutative-geometry setting. Section III assumes the power-law form  $F(R) = aR^{n-1}$  and discusses several solutions corresponding to different values of  $n$ , including the special cases  $n = 1$  (Einstein gravity) and  $n = 2$  ( $R^2$  gravity). Section IV assumes a particular form of the shape function that also yields a viable solution. Common to all these solutions is the absence of tidal forces.

## II. FIELD EQUATIONS IN $F(R)$ GRAVITY

To describe a spherically symmetric wormhole spacetime, we take the metric to be

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2)$$

Here  $\Phi(r)$  is a gravitational redshift function and  $b(r)$  is the shape function. The most general energy-momentum tensor is given by [16]

$$T_\nu^\mu = (\rho + p_r)u^\mu u_\nu - p_r g_\nu^\mu + (p_t - p_r)\eta^\mu \eta_\nu, \quad (3)$$

where  $u^\mu u_\mu = -\eta^\mu \eta_\mu = 1$  and  $u^\mu \eta_\mu = 0$ . Here the vector  $u^\mu$  is the fluid 4-velocity and  $\eta^\mu$  is a space-like vector orthogonal to  $u^\mu$ .

According to Ref. [16], the gravitational field equations in  $f(R)$  gravity can be written as

$$\rho(r) = \frac{Fb'}{r^2}, \quad (4)$$

$$p_r(r) = -\frac{Fb}{r^3} + \frac{F'}{2r^2}(b'r - b) - F''\left(1 - \frac{b}{r}\right), \quad (5)$$

$$p_t(r) = -\frac{F'}{r}\left(1 - \frac{b}{r}\right) - \frac{F}{2r^3}(b'r - b). \quad (6)$$

The above equations are the generic expressions for the matter threading the wormhole as a function of the shape function, as well as the specific form of  $F(r)$ , where  $F = \frac{df}{dR}$ . Here the prime denotes the derivative with respect to  $r$ .

The curvature scalar  $R$  is given by

$$R(r) = 2\frac{b'}{r^2}. \quad (7)$$

## III. WORMHOLES FOR GIVEN $F(R)$ FUNCTIONS

The past several decades have seen an immense interest by theorists in searching for viable alternative theories of modified gravity. The basic intent in modifying or extending general relativity was to explain certain cosmic phenomena such as dark matter, cosmic inflation in the early Universe, and the present cosmic accelerated expansion [17]. Furthermore “it is of interest to study gravitational theories which are diffeomorphism invariant and give Einstein gravity in an appropriate limit, but deviate from Einstein gravity in some way outside of the realm where gravitational effects have commonly been observed [19].” Our interest is confined to  $f(R)$  gravity, where  $R$  is the Ricci scalar. This gravitational theory has attained several theoretical and observational triumphs in recent years. (For reviews see [18] and references therein.)

We are going to assume a constant redshift function for our model, the so-called zero-tidal force solution, i.e.,  $\Phi(r) = \Phi_0$  (where  $\Phi_0$  is a constant) and a power-law form

$$F(R) = aR^{n-1}. \quad (8)$$

Here  $a$  is a constant and  $n$  is an integer. It is worth noting that wormholes with power-law  $F(R)$  gravity and a non-constant redshift function have been explored in the literature [19], but for simplicity and viability purposes, we shall restrict ourselves to the above assumption.

In solving the field equations, we have taken the energy density of the static and spherically symmetric smeared and particle-like gravitational source to be of the form given in Eq. (1):

$$\rho(r) = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}}; \quad (9)$$

as already noted, the mass  $M$  could be a diffused centralized object such as a wormhole.

We obtain the shape function  $b(r)$  by solving the differential equation obtained from Eqs. (4), (7), (8), and (9). The result is

$$b(r) = m_0 \left[ -2rn\theta e^{-\frac{r^2}{4n\theta}} + 2n^{\frac{3}{2}}\theta^{\frac{3}{2}}\pi^{\frac{1}{2}} \operatorname{erf} \left\{ \frac{r}{2\sqrt{n\theta}} \right\} + C \right], \quad (10)$$

where

$$m_0 = \left( \frac{M}{2^{n-1}a(4\pi\theta)^{\frac{3}{2}}} \right)^{\frac{1}{n}},$$

$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ , the error function, and  $C$  is an integration constant. It is easily checked that  $R$  becomes

$$R = 2 \left( \frac{M}{a2^{n-1}(4\pi\theta)^{\frac{3}{2}}} \right)^{\frac{1}{n}} e^{-\frac{r^2}{4n\theta}}. \quad (11)$$

In Eq. (10),  $n = 1$  corresponds to Einstein gravity, also obtained in Ref. [14]; the case  $n = 2$  is commonly referred to as quadratic or  $R^2$  gravity.

To get the exact physical characteristics, we will discuss several models resulting from different choices of  $n$ .

#### Subcase I: $n = 1$

By Eq. (8), the assumption  $n = 1$  implies that we are dealing with Einstein gravity with a noncommutative-geometry background. Here the shape function assumes the form

$$b(r) = m_0 \left[ -2r\theta e^{-\frac{r^2}{4\theta}} + 2\theta^{\frac{3}{2}}(\pi)^{\frac{1}{2}} \operatorname{erf} \left\{ \frac{r}{2\sqrt{\theta}} \right\} + C \right], \quad (12)$$

where

$$m_0 = \frac{M}{a(4\pi\theta)^{\frac{3}{2}}}.$$

This result agrees with our earlier result [14].

#### Subcase II: $n = 2$

The assumption  $n = 2$  corresponds to  $R^2$  gravity with a noncommutative-geometry background. Now the shape function takes on the form

$$b(r) = m_0 \left[ -4r\theta e^{-\frac{r^2}{8\theta}} + 4\theta^{\frac{3}{2}}(2\pi)^{\frac{1}{2}} \operatorname{erf} \left\{ \frac{r}{2\sqrt{2\theta}} \right\} + C \right], \quad (13)$$

where

$$m_0 = \left( \frac{M}{2a(4\pi\theta)^{\frac{3}{2}}} \right)^{\frac{1}{2}}.$$

The next step is to check that the shape function leads to the required wormhole structure. Using some typical values of the parameters, the resulting shape function is pictured in Fig. 1 (left panel). The middle panel shows that  $b(r)/r \rightarrow 0$  as  $r \rightarrow \infty$ ; so if the constant redshift function is joined smoothly to a function going to zero as  $r \rightarrow \infty$ , the spacetime becomes asymptotically flat. The throat of the wormhole is located at  $r = 0.35$ , where  $G(r) = b(r) - r$  cuts the  $r$ -axis, shown in Fig. 1 (right panel), also born out by Fig. 1 (left panel). In addition, Fig. 1 indicates that for  $r > r_0$ ,  $G(r) < 0$ , i.e.,  $b(r) - r < 0$ , which implies that  $\frac{b(r)}{r} < 1$  for  $r > r_0$ , an essential requirement for a shape function. Also,  $G(r)$  is a decreasing function for  $r > r_0$ . Since  $G'(r) < 0$ , we have  $b'(r_0) < 1$ , which is the flare-out condition. It now becomes apparent that the shape function has produced the desired wormhole structure.

It is interesting to note that on a cosmological scale, quadratic  $R^2$  gravity is considered physically viable since it is renormalizable and has numerous astrophysical implications [20].

The radial pressure component is given by

$$p_r(r) = -\frac{2am_0e^{-\frac{r^2}{8\theta}}}{r^3} \left[ m_0 \left\{ -4r\theta e^{-\frac{r^2}{8\theta}} + 4\theta^{\frac{3}{2}}(2\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{2\theta}} \right) + C \right\} \right. \\ \left. - \frac{am_0e^{-\frac{r^2}{8\theta}}}{2\theta r} \left[ m_0r^3e^{-\frac{r^2}{8\theta}} - m_0 \left\{ -4r\theta e^{-\frac{r^2}{8\theta}} + 4\theta^{\frac{3}{2}}(2\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{2\theta}} \right) + C \right\} \right] \right. \\ \left. - \left[ -\frac{am_0e^{-\frac{r^2}{8\theta}}}{2\theta} + \frac{am_0r^2e^{-\frac{r^2}{8\theta}}}{8\theta^2} \right] \left[ 1 - \frac{m_0}{r} \left\{ -4r\theta e^{-\frac{r^2}{8\theta}} + 4\theta^{\frac{3}{2}}(3\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{2\theta}} \right) + C \right\} \right] \right] \quad (14)$$

and the transverse pressure component is

$$p_t(r) = \left[ \frac{am_0e^{-\frac{r^2}{8\theta}}}{2\theta} \right] \left[ 1 - \frac{m_0}{r} \left\{ -4r\theta e^{-\frac{r^2}{8\theta}} + 4\theta^{\frac{3}{2}}(2\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{2\theta}} \right) + C \right\} \right] \\ - \frac{am_0e^{-\frac{r^2}{8\theta}}}{r^3} \left[ m_0r^3e^{-\frac{r^2}{8\theta}} - m_0 \left\{ -4r\theta e^{-\frac{r^2}{8\theta}} + 4\theta^{\frac{3}{2}}(2\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{2\theta}} \right) + C \right\} \right]. \quad (15)$$

Fig. 2 (right panel) indicates that the null energy condition is violated. Fig. 2 (left panel) shows the derivative of

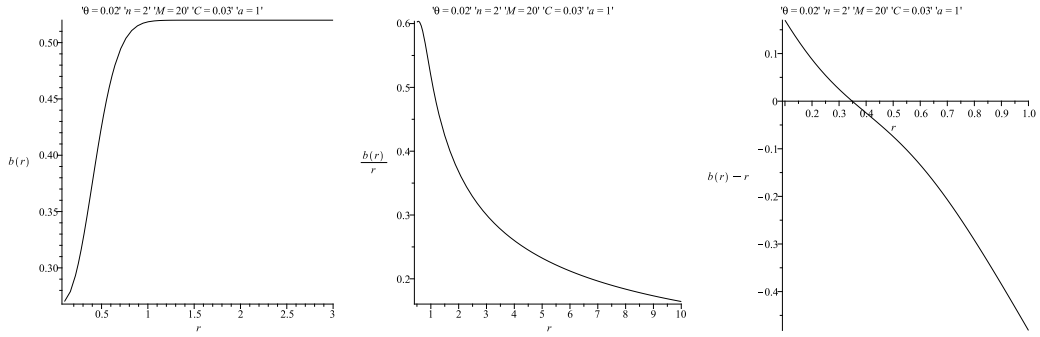


FIG. 1: (Left) Diagram of the shape function of the wormhole in  $R^2$  gravity for specific values of the parameters:  $\theta = 0.02$ ,  $M = 20$ ,  $a = 1$ , and  $C = 0.03$ . (Middle) Asymptotic behavior of the shape function. (Right) The throat occurs where  $G(r) = b(r) - r$  cuts the  $r$ -axis.

the shape function with respect to  $r$ .

### Subcase III: $n = 3$

The assumption  $n = 3$  means that we are dealing with  $R^3$  gravity with noncommutative geometry. Now the shape function becomes

$$b(r) = m_0 \left[ -6r\theta e^{-\frac{r^2}{12\theta}} + 6\theta^{\frac{3}{2}}(3\pi)^{\frac{1}{2}} \operatorname{erf} \left\{ \frac{r}{2\sqrt{3\theta}} \right\} + C \right], \quad (16)$$

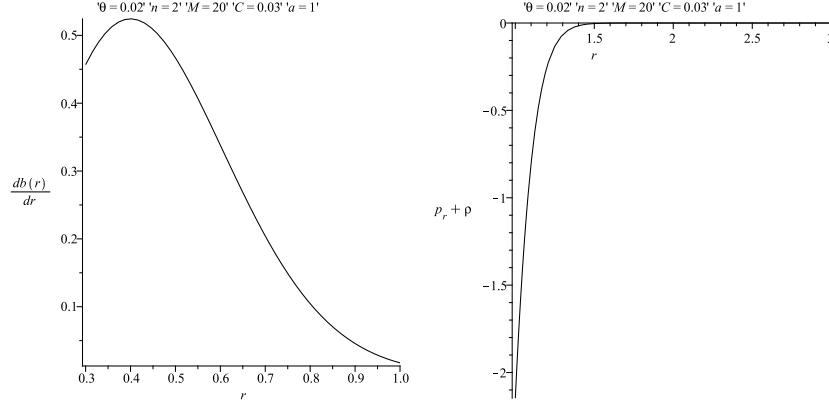


FIG. 2: (Left) Diagram of the derivative of the shape function of the wormhole. (Right) The variation of the left-hand side of the expression for the null energy condition with respect to  $r$ .

where

$$m_0 = \left( \frac{M}{4a(4\pi\theta)^{\frac{3}{2}}} \right)^{\frac{1}{3}}.$$

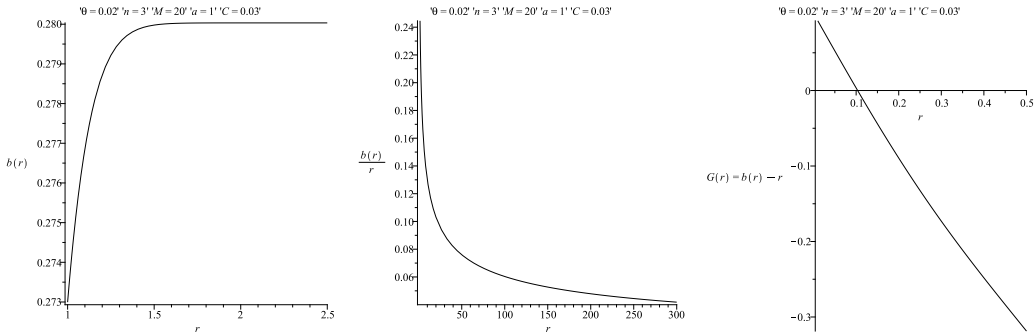


FIG. 3: (Left) Diagram of the shape function of the wormhole in  $R^3$  gravity for specific values of the parameters:  $\theta = 0.02$ ,  $M = 20$ ,  $a = 1$ , and  $C = 0.03$ . (Middle) Asymptotic behavior of the shape function. (Right) The throat occurs where  $G(r) = b(r) - r$  cuts the  $r$ -axis.

The radial pressure component is given by

$$\begin{aligned} p_r(r) = & -\frac{4am_0^2 e^{-\frac{r^2}{12\theta}}}{r^3} \left[ m_0 \left\{ -6r\theta e^{-\frac{r^2}{12\theta}} + 6\theta^{\frac{3}{2}}(3\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{3\theta}} \right) + C \right\} \right] \\ & - \frac{4am_0^2 e^{-\frac{r^2}{6\theta}}}{6\theta r} \left[ m_0 r^3 e^{-\frac{r^2}{12\theta}} - m_0 \left\{ -6r\theta e^{-\frac{r^2}{12\theta}} + 6\theta^{\frac{3}{2}}(3\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{3\theta}} \right) + C \right\} \right] \\ & - \left[ -\frac{4am_0^2 e^{-\frac{r^2}{6\theta}}}{3\theta} + \frac{4am_0^2 r^2 e^{-\frac{r^2}{6\theta}}}{9\theta^2} \right] \left[ 1 - \frac{m_0}{r} \left\{ -6r\theta e^{-\frac{r^2}{12\theta}} + 6\theta^{\frac{3}{2}}(3\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{3\theta}} \right) + C \right\} \right], \end{aligned} \quad (17)$$

while the transverse pressure component is

$$\begin{aligned} p_t(r) = & \left[ \frac{4am_0^2 e^{-\frac{r^2}{6\theta}}}{3\theta} \right] \left[ 1 - \frac{m_0}{r} \left\{ -6r\theta e^{-\frac{r^2}{12\theta}} + 6\theta^{\frac{3}{2}}(3\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{3\theta}} \right) + C \right\} \right] \\ & - \frac{2am_0^2 e^{-\frac{r^2}{6\theta}}}{r^3} \left[ m_0 r^3 e^{-\frac{r^2}{12\theta}} - m_0 \left\{ -6r\theta e^{-\frac{r^2}{12\theta}} + 6\theta^{\frac{3}{2}}(3\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{3\theta}} \right) + C \right\} \right]. \end{aligned} \quad (18)$$

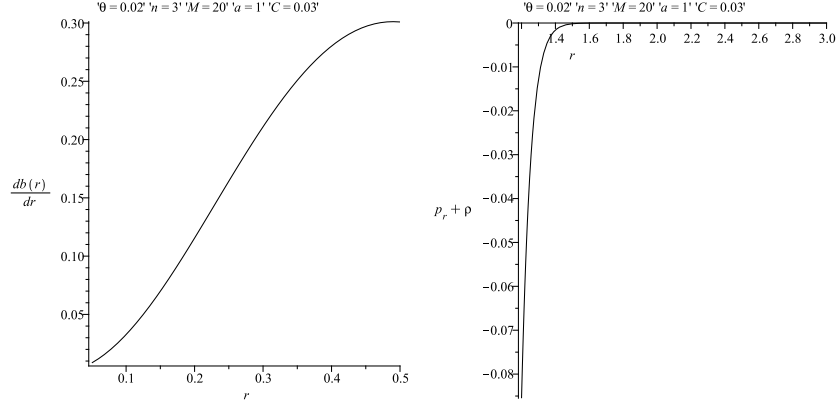


FIG. 4: (Left) Diagram of the derivative of the shape function of the wormhole. (Right) The variations of the left-hand side of the expression for the null energy condition with respect to  $r$ .

Fig. 3 illustrates all the necessary characteristics of the shape function of wormhole, while Fig. 4, in addition to the derivative of the shape function, shows that the null energy condition is violated.

#### Subcase IV: $n = 4$

The assumption  $n = 4$  yields  $R^4$  gravity with noncommutative geometry. Here the shape function, radial pressure, and transverse pressure take on the respective forms

$$b(r) = m_0 \left[ -8r\theta e^{-\frac{r^2}{16\theta}} + 8\theta^{\frac{3}{2}}(4\pi)^{\frac{1}{2}} \operatorname{erf} \left\{ \frac{r}{2\sqrt{4\theta}} \right\} + C \right], \quad (19)$$

where

$$m_0 = \left( \frac{M}{8a(4\pi\theta)^{\frac{3}{2}}} \right)^{\frac{1}{4}}$$

$$\begin{aligned} p_r = & -\frac{8am_0^2 e^{-\frac{3r^2}{16\theta}}}{r^3} \left[ m_0 \left\{ -8r\theta e^{-\frac{r^2}{16\theta}} + 8\theta^{\frac{3}{2}}(4\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{4\theta}} \right) + C \right\} \right. \\ & \left. - \frac{3am_0^3 e^{-\frac{3r^2}{16\theta}}}{2\theta r} \left[ m_0 r^3 e^{-\frac{r^2}{16\theta}} - m_0 \left\{ -8r\theta e^{-\frac{r^2}{16\theta}} + 8\theta^{\frac{3}{2}}(4\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{4\theta}} \right) + C \right\} \right] \right. \\ & \left. - \left[ -\frac{3am_0^3 e^{-\frac{3r^2}{16\theta}}}{\theta} + \frac{9am_0^3 r^2 e^{-\frac{3r^2}{16\theta}}}{8\theta^2} \right] \left[ 1 - \frac{m_0}{r} \left\{ -8r\theta e^{-\frac{r^2}{16\theta}} + 8\theta^{\frac{3}{2}}(4\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{4\theta}} \right) + C \right\} \right] \right], \end{aligned} \quad (20)$$

$$\begin{aligned} p_t(r) = & \left[ \frac{3am_0^3 e^{-\frac{3r^2}{16\theta}}}{\theta} \right] \left[ 1 - \frac{m_0}{r} \left\{ -8r\theta e^{-\frac{r^2}{16\theta}} + 8\theta^{\frac{3}{2}}(4\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{4\theta}} \right) + C \right\} \right] \\ & - \frac{4am_0^3 e^{-\frac{3r^2}{16\theta}}}{r^3} \left[ m_0 r^3 e^{-\frac{r^2}{16\theta}} - m_0 \left\{ -8r\theta e^{-\frac{r^2}{16\theta}} + 8\theta^{\frac{3}{2}}(4\pi)^{\frac{1}{2}} \operatorname{erf} \left( \frac{r}{2\sqrt{4\theta}} \right) + C \right\} \right]. \end{aligned} \quad (21)$$

As before, Figs. 5 and 6 illustrate the various characteristics of the wormhole.

**Summary of Section III:** The power-law form of the modified gravity,  $F(R) = aR^{n-1}$ , with a noncommutative-geometry background yielded wormhole solutions with very similar characteristics, including the violation of the null energy condition. The most striking feature is a decreasing throat size as the power of  $F(R)$  increases.

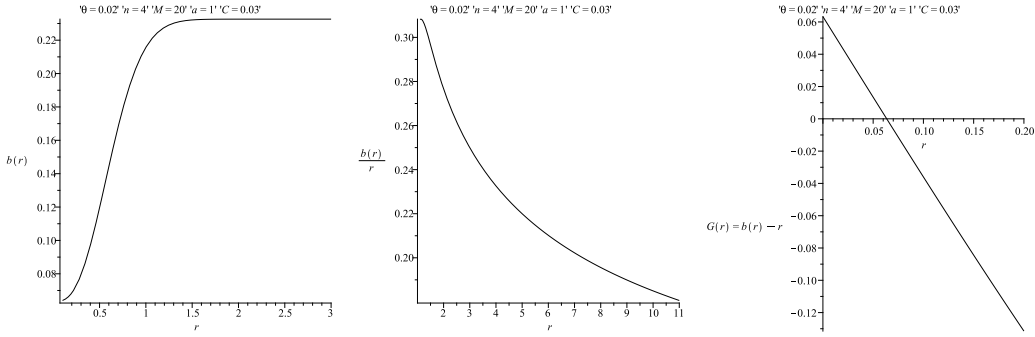


FIG. 5: (Left) Diagram of the shape function of the wormhole in  $R^4$  gravity for specific values of the parameters:  $\theta = 0.02$ ,  $M = 20$ ,  $a = 1$ , and  $C = 0.03$ . (Middle) Asymptotic behavior of the shape function. (Right) The throat occurs where  $G(r) = b(r) - r$  cuts the  $r$ -axis.

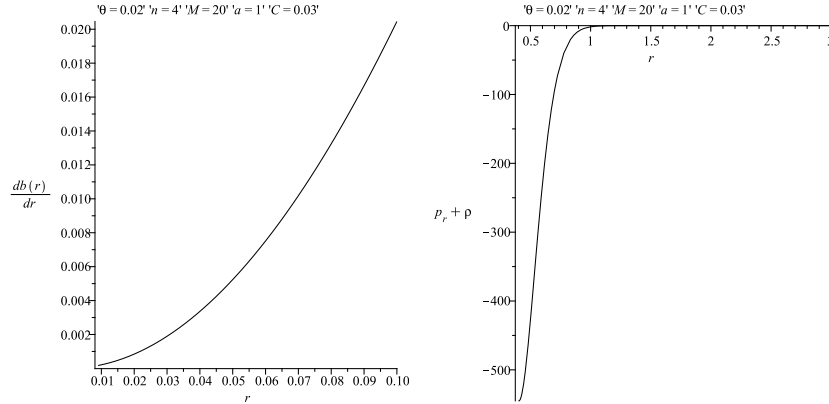


FIG. 6: (Left) Diagram of the derivative of the shape function of the wormhole. (Right) The variation of the left-hand side of the expression for the null energy condition with respect to  $r$ .

#### IV. WORMHOLE SOLUTION FOR A GIVEN SHAPE FUNCTION

This section discusses another wormhole solution using a particular shape function [2]. For this case, the null energy condition is violated but the strong energy condition is met. The function is

$$b(r) = r_0 \left( \frac{r}{r_0} \right)^\alpha, \quad \alpha < 1. \quad (22)$$

Observe that  $b(r_0) = r_0$  and  $b'(r) = \alpha < 1$ , so that the flare-out condition is satisfied. From Eq. (4) we obtain

$$F(r) = \frac{\rho r^2}{b'(r)} = \frac{\rho r^{3-\alpha}}{\alpha r_0^{1-\alpha}}. \quad (23)$$

Moreover, the curvature scalar is given by

$$R = 2\alpha \frac{r^{\alpha-3}}{r_0^{\alpha-1}} < \infty. \quad (24)$$

So the curvature scalar is finite since  $r_0 > 0$ . Using the above expressions, it is easy to reconstruct  $F(R)$ :

$$F(R) = \frac{R}{2\alpha^2} \exp \left[ -\frac{1}{4\theta} \left\{ \frac{R}{2\alpha} \left( \frac{2\alpha}{R_0} \right)^{2(1-\alpha)} \right\}^{\frac{2}{3-\alpha}} \right]; \quad (25)$$

here  $R_0$  is the value of curvature scalar at  $r = r_0$ .

Making use of Eqs. (22) and (23) in (5) and (6), we get the radial pressure

$$\begin{aligned}
p_r = & \frac{M}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}} \left[ -\frac{r^\alpha}{3\alpha} + \frac{r^{4-2\alpha}}{r_0^{2(1-\alpha)} 4\alpha^2(\alpha-3)} \left( -\frac{r^2}{2\theta} + 3 - \alpha \right) r_0^{1-\alpha} r^\alpha (\alpha-1) \right. \\
& - \frac{r^{9-3\alpha}}{2\alpha(\alpha-3) r_0^{1-\alpha}} \left\{ -\frac{r^2}{4\theta r_0^{2(1-\alpha)} \alpha^2(\alpha-3)} \left( -\frac{r^2}{2\theta} + 3 - \alpha \right) \right. \\
& \left. \left. + \frac{1}{r_0^2(\alpha-1)} \left( -\frac{r^2}{2\theta} + 3 - \alpha \right) + \frac{r}{r_0^{2(1-\alpha)} 2\alpha^2(\alpha-3)} \left( -\frac{r}{\theta} + 3 - \alpha \right) \right\} (1 - r_0^{1-\alpha} r^{\alpha-1}) \right] \quad (26)
\end{aligned}$$

as well as the transverse pressure

$$\begin{aligned}
p_t(r) = & \frac{M}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}} \left[ -\frac{r^{5-2\alpha}}{r_0^{2(1-\alpha)} 2\alpha^2(\alpha-3)} \left( -\frac{r^2}{2\theta} + 3 - \alpha \right) (1 - r_0^{1-\alpha} r^{\alpha-1}) \right. \\
& \left. + \frac{r^{3-\alpha}}{r_0^{1-\alpha} 4\alpha^2(\alpha-3)} \left( -\frac{r^2}{2\theta} + 3 - \alpha \right) (1 - \alpha) \right]. \quad (27)
\end{aligned}$$

The physical viability of this solution is determined by checking the energy conditions. We can see from Fig. 7 that  $p_r + \rho < 0$ . It is interesting to note the strong energy condition is satisfied.

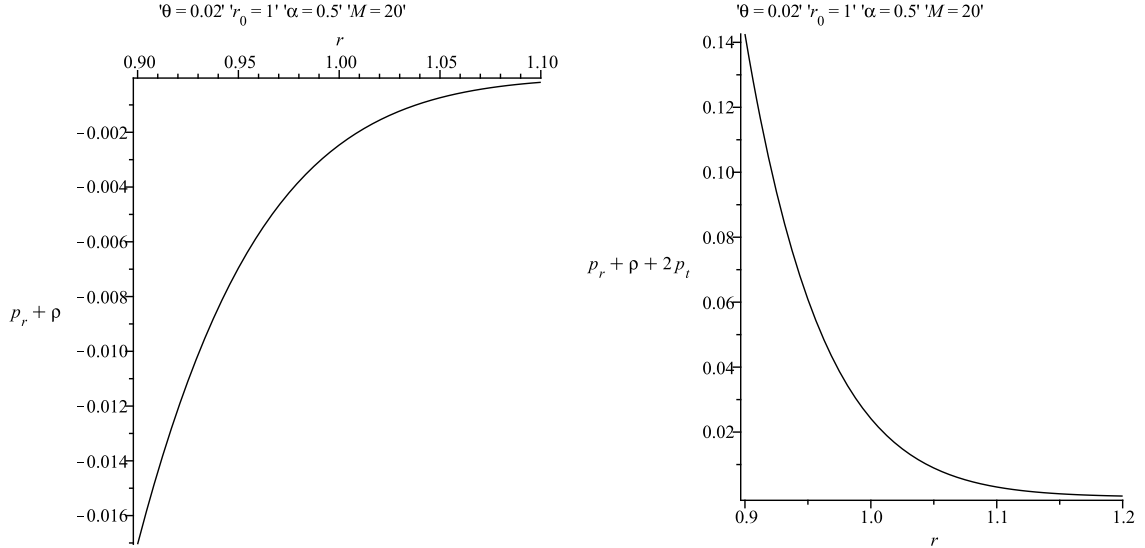


FIG. 7: (Left) The variation of the radial null energy conditions with respect to  $r$ . (Right) The behavior of the strong energy condition is shown against  $r$ .

## V. CONCLUSION

Noncommutative geometry, an offshoot of string theory, replaces pointlike structures by smeared objects and has recently been extended to higher dimensions. In this paper we present two models of wormholes within the framework of this extended noncommutative geometry. The first model assumes the power-law form  $F(R) = aR^{n-1}$ . The analysis includes the important special cases  $n = 1$  (Einstein gravity), already discussed in Ref. [14], and  $n = 2$  ( $R^2$  gravity). It is shown that the basic characteristics, particularly the violation of the null energy condition, remain essentially the same, but the radius of the throat decreases as the power of  $F(R)$  increases. The second model discussed assumes a particular shape function, thereby allowing a reconstruction of  $F(R)$ . For this case, the null energy condition is once again violated, but the strong energy condition is met. All the solutions assume zero tidal forces, a highly desirable feature from the standpoint of wormhole design.



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